Announcements: Homework #1

- Due Friday at 3PM outside Math 113 in the box labeled "Section 204"
- If you used LaTeX, you can email to me, but do so by 2PM ksankar@math.
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Last time: Truth tables

P	Q	$P \Rightarrow Q$	
T	T	T	⟵ Ex. n=6
T	F	F	
F	T	T	⟵ Ex. n=4
F	F	T	⟵ Ex. n=5

Ex: If n is a multiple of 6 then n is even

Note: $\underbrace{(\sim P)}_{\text{P is false}} \vee \underbrace{Q}_{\text{Q is true.}}$ is equivalent to $(P \Rightarrow Q)$

Exercise/Note: $Q \Rightarrow P$ (converse of $P \Rightarrow Q$) $\equiv (\sim Q) \vee P$

Try these truth tables! $\sim P \Rightarrow \sim Q \quad \equiv (\sim\sim P) \vee (\sim Q)$

Contrapositive $[\sim Q \Rightarrow \sim P \equiv (\sim\sim Q) \vee (\sim P) \equiv (P \Rightarrow Q)$

$Q \vee \sim P$

Proofs: Most statements can be written as

$$P \Rightarrow Q$$

Ex: "The square of an odd integer is odd."

~~or~~ $\equiv$ "If $\underbrace{n \text{ is an odd integer}}_{P}$, then $\underbrace{n^2 \text{ is odd}}_{Q}$."

$\equiv$ "Let n be an integer. If n is odd then n^2 is odd"

Ex: "$\sqrt{2}$ is irrational"

$\equiv$ "If x is a rational number, then $x^2 \neq 2$."

Definitions: It's useful to give precise definitions for concepts. **Ex:** "odd" "irrational" "prime"

Def: An integer n is "odd" if there is some integer a $\underbrace{\text{s.t.}}_{\text{such that}}$ $n = 2a+1$

Def: ______________ "even" ______

______ ________ $n = 2a$.

(Direct Proof)

Proposition: Let n be an integer.

If n is odd, then $3n+5$ is even.

Proof: n is odd. Therefore there is some $a \in \mathbb{Z}$ such that $n = 2a+1$.

So $\boxed{3n+5} = 3(2a+1)+5$

$= 6a+3+5 = 6a+8 = \boxed{2(3a+4)}$

Because $3a+4$ is an integer, $\boxed{2(3a+4)}$ is even

Therefore, $\boxed{3n+5}$ is even. ■

Exercise: If n is odd then n^2-5n+2 is even.

Exercise: If n is even then n^2-5n+2 is even.

Q: How to prove "For any integer n,

n^2-5n+2 is even."

(Proof by cases)

Next time)

Proposition: If n is odd then n^2-5n+2 is even

Pf: n is odd. Therefore there is some $a \in \mathbb{Z}$ s.t. $n = 2a+1$.

$$\begin{aligned} n^2-5n+2 &= (2a+1)^2 - 5(2a+1) + 2 \\ &= 4a^2+4a+1-(10a+5)+2 \\ &= 4a^2-6a-2 \\ &= 2(2a^2-3a-1) \end{aligned}$$

Since $2a^2-3a-1$ is an integer, $2(2a^2-3a-1)$ is even.
Therefore, n^2-5n+2 is even. ■

Proposition: If n is even then n^2-5n+2 is even

Pf: n is even. Therefore there is some $b \in \mathbb{Z}$ s.t. $n = 2b$.

$$\begin{aligned} n^2-5n+2 &= (2b)^2 - 5(2b) + 2 \\ &= 4b^2 - 10b + 2 \\ &= 2(2b^2-5b+1) \end{aligned}$$

Since $2b^2-5b+1$ is an integer, $2(2b^2-5b+1)$ is even
Therefore, n^2-5n+2 is even. ■